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Energy spectrum and angular distribution of cosmic ray muons in the range 50–70 GeV

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Abstract. The energy dependence and angular distribution of the sea level muon intensity have been determined from measurements made inside the second pyramid at Giza, Cairo. It is concluded from these measurements that the sea level muon integral spectrum may be represented in the energy range 50–70 GeV by the power law

$$I(E, \theta) = KE^{-2.09}(\cos\theta)^{-0.02}$$

where E is expressed in GeV, θ is the angle to the vertical and K is a constant.

The above values of energy and cosine exponents are found to be in agreement with those calculated using the CKP interaction model if pions are assumed to be the parents of the majority of muons reaching sea level with energies in the range considered. Calculations made for various energies using this model also show general accordance with the experimental results of other authors. This indicates further support for the application of the model to primary interactions at high energies.

1. Introduction

Various experiments have been performed to study the characteristics of the muon integral spectrum at high energies using underground measurements. A recent review of such experiments has been given by Stockel (1969). Besides observations in mines under comparatively flat terrains, muon intensities have also been measured in tunnels underlying mountains. Examples of such measurements are those obtained under 163 mwe of rock in a deep tunnel in the Snowy Mountains of Australia (George 1955), under Monte Blanc in Italy at depths between 40 and 4100 mwe (Castagnoli *et al* 1965) and under mountainous overburden in Utah, USA at inclined depths ranging between 2000 and 8000 mwe (Bergeson *et al* 1967, 1968 and 1969).

In the present work it was possible to use the rock overlying the burial chamber in the second pyramid at Giza, Cairo, as a variable thickness absorber to select high energy muons. In the course of a joint project with the Lawrence Radiation Laboratory, Berkeley, USA and Ein Shams University, UAR, to search for unknown cavities in that pyramid, a plastic scintillator–spark chamber telescope has been used to record the directional intensity of muons penetrating to the pyramid burial chamber. The results of the experiment performed to probe the pyramid interior are reported elsewhere (Alvarez *et al* 1970).

The telescope has been installed near the south-east corner of the burial chamber which is located near the pyramid base centre (figure 1). The pyramid geometry and the opening angle of the telescope, which is about 80°, allow the measurements of muon intensities in zenith angles up to 40° in the range of energies between 50 and 70 GeV.

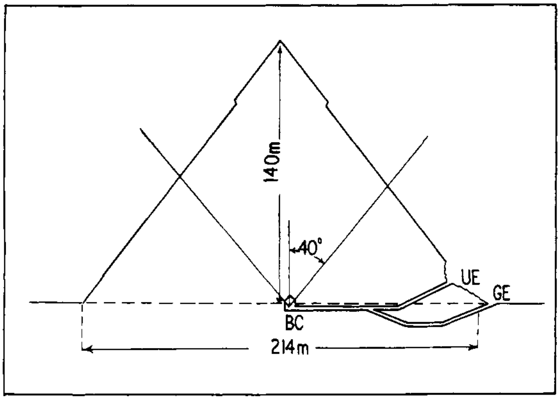

Figure 1. Section in Chephren Pyramid showing location of telescope. GE, ground entrance; UE, upper entrance; BC, burial chamber.

2. Experimental arrangement

Figure 2 shows a schematic representation of the telescope which is a combination of 14 plastic scintillators 1×1 m^2 each and four magnetostrictive wire spark chambers 1×2 m^2 each. The spark chambers are arranged in two layers, 30 cm apart, in between two layers of scintillators with four counters in each layer, providing an angular resolution of about 0·2°. The resulting four layers are arranged on top of a 36 ton iron absorber of thickness 1·2 m, under which a third layer of six scintillators is placed at floor level.

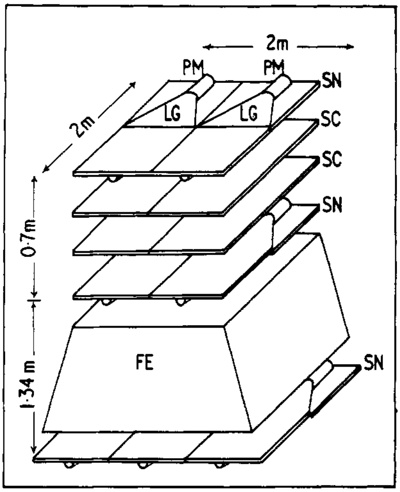

Figure 2. The cosmic ray telescope. LG, light guide; PM, photomultiplier; SN, scintillator layer; SC, spark chamber layer; FE, iron absorber.

The triple coincidence signal resulting from the passage of a muon through the system is used to trigger the spark chambers and the associated electronics. The x and y coordinates of the sparks in each spark chamber layer are digitized by a magnetostrictive read-out system of the type described by Perez Mendez and Pfab (1965). The digitized data are transferred through cables to a laboratory about 500 m away from the pyramid where they are recorded on magnetic tape using an IBM-7330 tape drive.

The raw data collected during a period of about four months consisted of two million events recorded on magnetic tapes. These were first filtered, using an IBM-1130 computer, to reject events not satisfying certain conditions, thus ensuring near perfect operation of the system. The rejected events amount to an average of 40 % of the raw data and they include primarily those events accompanied by misfiring or occurrence of multisparks in one or both spark chamber layers.

3. Experimental angular distribution of muons

The coordinates of the sparks in the two spark chamber layers have been used to compute the polar angles θ (zenithal) and ϕ (azimuthal) of the incident muon trajectories. All particles recorded within $0 < \theta < 40°$ and $0 < \phi < 360°$ have been distributed into a matrix of 20×180 bins, each two by two degrees. Examples of the azimuthal distributions for zenithal angles $\theta = 9°$ and 25° are shown by the experimental points in figure 3, taking azimuthal intervals $\Delta\phi = 8°$.

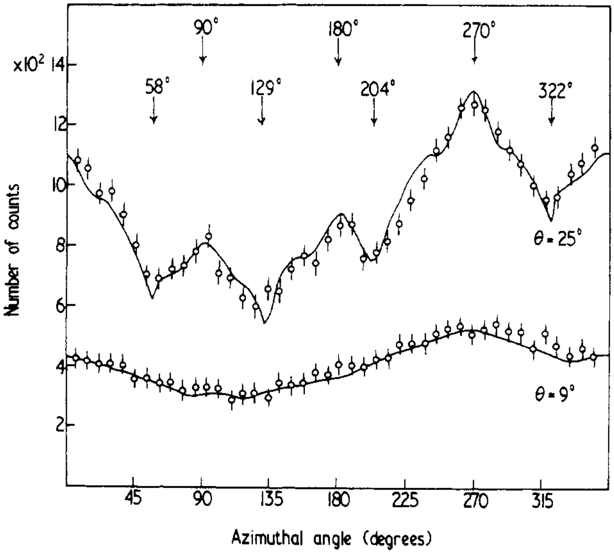

Figure 3. Experimental and theoretical azimuthal distributions of muon counts.

The four maxima and four minima appearing in the distribution for $\theta = 25°$ correspond to the geographical axes and diagonal ridges of the pyramid, respectively. For $\theta = 9°$, the distinction between maxima and minima is not as well marked as in the previous case since, inside the pyramid, the variation of muon range with respect to ϕ decreases as the zenith angle decreases.

The differences between maxima along the same geographical axis as well as the departure of positions of minima from multiples of $\pi/4$ indicate that the detector is displaced towards east and north from the projection of the pyramid apex on its base.

It was thus possible from cosmic ray data alone, to determine the accurate location of the detector (Alvarez *et al* 1970). The cosmic ray derived position (5 m north and 13·5 m east of the centre of the base) is in good agreement with a recently surveyed position obtained by the UAR surveying department.

4. Determination of muon spectrum parameters

Over a wide range of energies, the muon sea level integral energy spectrum may be represented by a power law of the form

$$I(E, \theta) = KE^{-m} \cos^n\theta$$

where the spectral index m, the cosine exponent n and K are slowly varying functions of threshold energy E.

The experimental data collected in the form of a θ, ϕ bin matrix have been used to determine the parameters m and n in the range of threshold energies 50–70 GeV. The procedure followed in this respect utilizes χ^2 fitting of the expected count distribution to the experimental one using the above equation and determining the values of m and n giving best fit.

The expected number of counts in a bin in a direction θ, ϕ and of size $\Delta\theta$, $\Delta\phi$ has been calculated using the equation

$$N(\theta, \phi) = A(R(\theta, \phi))^{-m} \cos^n\theta \; G(\theta, \phi) \sin\theta \; \Delta\theta \; \Delta\phi$$

where $R(\theta, \phi)$ is the muon range inside the pyramid, assumed to vary linearly with energy and $G(\theta, \phi)$ is the acceptance function of the telescope. In terms of geometrical distances, $R(\theta, \phi)$ can be expressed as follows:

$$R(\theta, \phi) = d + r_i - r_a \pm r_s$$

where d is the path length in the pyramid volume, assuming a completely smooth pyramid surface, r_i and r_a are the rock equivalent muon paths in the iron absorber and other material around the detector, respectively, and r_s is a correction due to pyramid surface irregularities. This last correction has been obtained from aerial photographs of the pyramid provided by the UAR surveying department and contributes to the total muon range by a maximum of 3%.

In fitting the expected muon count distribution $N(\theta, \phi)$ to the experimental one, the values of m and n giving minimum χ^2 were found to be $m = 2{\cdot}09 \pm 0{\cdot}04$, $n = -0{\cdot}02 \pm 0{\cdot}04$.

The azimuthal distributions at $\theta = 9°$ and $25°$ calculated using the above values of m and n are shown by the full curves in figure 3.

5. Derivation of theoretical energy spectrum and angular distribution of muons

A theoretical description of the sea level muon spectrum can be achieved by assuming a model for the primary interaction mechanism and adopting an expression for the primary proton energy spectrum. The relation given by Cocconi *et al* (1961) (the CKP relation) for the energy spectrum of generated pions has been adopted here. As a starting point for the energy spectrum of the primary protons, the expression given by Barrett *et al* (1952) has been used. This 'trial' expression is derived from a variety of experimental data and is simply used as a datum in the realization that the parameters may need to be modified later. The relation for the differential energy spectrum is

$$D(E)\,\mathrm{d}E = 0{\cdot}3\,SP^{S}(E+P)^{-(S+1)}\,\mathrm{d}E \qquad (\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}\,\mathrm{GeV}^{-1})$$

where S is a function of primary energy and is given by

$$S = 1{\cdot}35 + 0{\cdot}04\ln\left(1+\frac{E}{P}\right)$$

where $P = 3{\cdot}2$ and E is measured in GeV (the expression is valid for $10 < E < 10^5$ GeV). Over a restricted range of energies, the expression can be written as

$$D(E)\,\mathrm{d}E = BE^{-\gamma}\,\mathrm{d}E$$

where

$$\gamma = \frac{SE}{E+P} + 1$$

and this expression will be used in the present calculations.

5.1. *The production spectrum of pions*

The spectrum of pions at production has been derived essentially following the method outlined by Brooke *et al* (1964). The CKP relation used in this derivation expresses the number of pions emitted in the forward direction in the C system, as a result of the interaction of a primary having energy E with an air nucleus, in the form

$$N(E_\pi, E)\,\mathrm{d}E_\pi = \frac{A}{T}\exp\left(-\frac{E_\pi}{T}\right)\mathrm{d}E_\pi$$

where E_π is the energy of the pion in the L system, A is the multiplicity of pions emitted in the forward direction in the C system, assumed to vary as E^α where $\alpha = \frac{1}{4}$, and T is the mean pion energy. The pion production spectrum is then given by

$$F(E_\pi)\,\mathrm{d}E_\pi = \frac{2}{1-(1-K_\mathrm{t})^{\gamma-1}}\,\mathrm{d}E_\pi \int_{3E}^{\infty} N(E_\pi, E) D(E)\,\mathrm{d}E$$

where the term $1/\{1-(1-K_\mathrm{t})^{\gamma-1}\}$ gives the sum of contributions of various production generations and K_t is the primary proton inelasticity. Evaluation of the integral then gives

$$\begin{aligned} F(E_\pi)\,\mathrm{d}E_\pi &= \left\{\frac{2}{1-(1-K_\mathrm{t})^{\gamma-1}}\,\frac{B}{1-\alpha}\,a^u\left(\frac{K_\pi}{3}\right)^{v} I(v+1, y)\right\} E_\pi^{-\gamma_\pi}\,\mathrm{d}E_\pi \\ &= A_\pi E_\pi^{-\gamma_\pi}\,\mathrm{d}E_\pi \end{aligned}$$

where

$$u = \frac{2-\gamma}{1-\alpha} \qquad v = \frac{\gamma-\alpha-1}{1-\alpha} \qquad \gamma_\pi = \frac{\gamma-2\alpha}{1-\alpha} \qquad y = \frac{a}{K_\pi}\,(3E_\pi)^\alpha$$

$a = 0{\cdot}45$ and K_π is the mean fraction of proton energy passed on to the pion component in each interaction. The term $I(v+1, y)$ expresses the incomplete Γ function

$$I(v+1, y) = \int_0^y x^v \mathrm{e}^{-x}\,\mathrm{d}x$$

and may be approximated to unity at moderate primary energies ($3E_\pi < E < 10^{12}$ eV) and high pion energies. In this case, the expression for $F(E_\pi)$ would be identical to that given by Brooke *et al* (1964).

The resulting values of γ_π and A_π are shown in figure 4 (denoted by C1), the expression of Barrett *et al* (1952) having been used to give the magnitudes of B and γ in the expression for the primary spectrum.

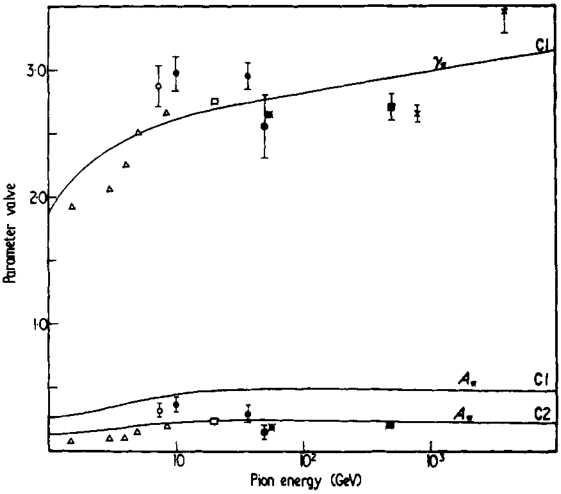

Figure 4. Pion production spectrum parameters. ——— present work; △ Gardener *et al* (1962); □ Puppi (1956); ■ Ashton and Wolfendale (1963); × Miyake *et al* (1964); ⊗ Pak *et al* (1961); ○ Coates and Nash (1962); ● Judge and Nash (1965a, 1965b); ⊙ Allen and Apostolakis (1961).

Also shown are values derived from experimental data on muon spectra by various authors, the method being to allow for μ–e decay and loss of pions by interaction in transforming from measured muon spectra to pion production spectra. From figure 4 it can be seen that the behaviour of γ_π is in general accordance with the experimental data. However, there is a marked discrepancy between experimental and calculated values of A_π, the latter being higher by a factor of about two at most energies.

There are a number of possible explanations for the discrepancy. One is to assume that the 'trial' expression for the primary spectrum overestimates the intensity in the relevant energy region and this is the explanation advanced by Brooke *et al* (1964). Others are to assume that the CKP model is not accurate here or that a significant fraction of the energy of the primary particles does not appear as pions.

More recent measurements of the primary intensity (by Malhotra *et al* 1966 and others) give a mean slope for the range 10–10^4 GeV not far from that of the trial spectrum but indicate intensities about a factor of two below the trial values and the effect on A_π of a reduction by this factor (figure 4, denoted by C2) clearly gives much better agreement.

Taking the agreement for values of γ_π together with the improved fit of the A_π values there is evidence for the applicability of the adopted interaction model, which comprises the CKP relation, a multiplicity law varying as $E^{1/4}$ and a small fraction of nonpions generated over a wide range of energy (the CKP relation originally put forward to account for nucleon interactions at machine energies ie < 30 GeV). Similar evidence has been indicated by Bowler *et al* (1962), Brooke *et al* (1964), Craig *et al* (1968) and Adcock *et al* (1969) from high energy γ ray and muon measurements. Of course there is no suggestion that the model is exact but rather that its essential features are correct.

One feature of this model—the nonexistence of a large kaon contribution to the muon intensity in the energy range considered—is in accordance with various results deduced from experiments made at large zenith angles in similar energy ranges (Ashton and Wolfendale 1963 and Ashton *et al* 1966).

5.2. *The parameters of the sea level muon integral spectrum*

Following Judge and Nash (1965a), the differential spectrum of sea level muons at zenith angle θ to the vertical is given by

$$I(E_\mu, \theta)\,\mathrm{d}E_\mu = \frac{1}{r_x} F_x\left(\frac{E_{\mu\mathrm{p}}}{r_x}\right) \frac{B_x W(E_{\mu\mathrm{p}}, \theta)}{B_x + E_{\mu\mathrm{p}} \cos\theta}\,\mathrm{d}E_\mu$$

where $E_{\mu\mathrm{p}}$ and E_μ are the muon energy at production and sea level respectively, $W(E_{\mu\mathrm{p}}, \theta)$ its survival probability and $F_x(E_{\mu\mathrm{p}}/r_x)$ the muon parent production spectrum. In the case of pion parentage $B_\pi = 90$ GeV and $r_\pi = 0{\cdot}787$. If such a spectrum is represented by a simple power law with exponent ϵ then the integral spectrum may be represented by the formula

$$I(E_\mu, \theta) = K E_\mu^{-m} \cos^n\theta .$$

The dependence of spectral index $m = \epsilon - 1$ and cosine exponent n on muon energy can be obtained in the form

$$m = -\frac{E_\mu}{I} \frac{\partial I}{\partial E_\mu} - 1 = \frac{E_\mu}{E_{\mu\mathrm{p}}}(\gamma_x + \delta_x - \delta_\mu) - 1$$

and

$$n = \frac{\cos\theta}{I} \frac{\partial I}{\partial \cos\theta} = \frac{E_\mu}{E_{\mu\mathrm{p}}}(\delta_\mu - \delta_x) + \frac{d}{E_{\mu\mathrm{p}} \cos\theta}(\gamma_x + \delta_x)$$

where the terms γ_x and δ_x express the production spectrum exponent and nuclear interaction probability of muon parents, respectively, while δ_μ expresses the change in spectrum due to muon decay and ionization losses in the atmosphere and d is the energy loss, by ionization, for a vertically incident muon.

Using the above equations, values of m and n have been calculated for the extreme cases of 100% pion and 100% kaon parentage assuming that the kaon production spectrum is identical to that of pions deduced from the CKP relation. The results of these calculations in the energy range 1–10^3 GeV are represented by the full curves in figures 5 and 6. Also shown in these figures (broken curves) are the calculations made by Budini and Molière (1953) applying a different model and using a constant value for the primary energy spectrum exponent.

Regarding the general behaviour of m and n in the energy region considered, it can be seen from figures 5 and 6 that fair agreement exists between present calculations and those of Budini and Molière. However, some differences are noted, especially in the effect of kaon parentage. In the case of both m and n this effect appears in the calculations of the latter authors only at energies greater than 150 GeV. Moreover, their calculations for n in the case of pion parentage do not produce the sec θ enhancement predicted by the present calculations for the muon angular distribution at high energies. Such enhancement should be expected in both cases of pion and kaon parentage since it is a consequence of the competition between decay and nuclear interaction of those particles at high energies.

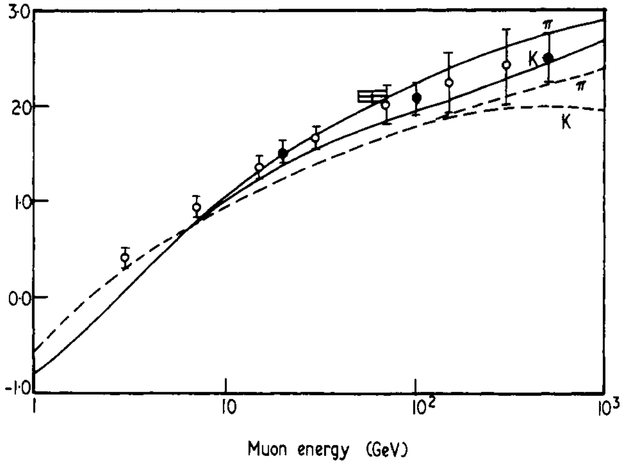

Figure 5. Comparison between observed and predicted values of m. ——— present theoretical curves; --- Budini and Molière (1953); ○ Hayman and Wolfendale (1962); ● Aurela and Wolfendale (1967); ⊞ present experimental result.

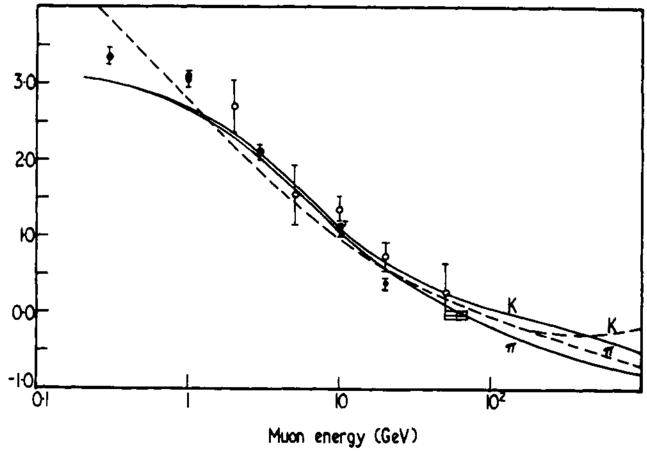

Figure 6. Comparison between observed and predicted values of n. ——— present theoretical curves; --- Budini and Molière (1953); ● Moroney and Parry (1954); ɸ Judge and Nash (1965a); ⊞ present experimental result.

6. Comparison of theory with experiment

The value of m obtained from the present experiment has been presented in figure 5 together with those deduced from the experimental results obtained by Hayman and Wolfendale (1962) from a measurement of the vertical differential muon spectrum using a magnetic spectrograph and the more recent analysis of Aurela and Wolfendale (1967). The experimental value is found to be in general accordance with results of these authors in the same energy region.

The values of m found in the other experiments are close to those predicted from the theoretical analysis given here, particularly in the energy range 10–100 GeV. Above this energy the form of the variation may be a little different but the experimental errors are too great to attach much importance to this fact.

In principle it is possible to determine the fraction of muons which derive from kaons using this analysis (which relates to the slope of the spectra, in contrast to the analysis in § 5 which refers more particularly to absolute intensities). However, experimental errors coupled with small uncertainties in the model preclude this.

In figure 6, the value of n determined from the present experiment is presented together with those obtained by Moroney and Parry (1954) and Judge and Nash (1965a). It is seen that the measured value is close to what would be expected at the energy in question from the other two experiments.

Concerning the theoretical analysis, there is reasonable agreement with experiment, particularly above several GeV. At lower energies, the use of a constant height of production of the muons breaks down and the prediction becomes increasingly unreliable.

In conclusion, the experimental values found in the present work are close to those found by previous authors and add weight to our knowledge of the near sea level muon spectrum and its variation with zenith angle. The theoretical analysis indicates that a simple model for cosmic ray propagation in the atmosphere gives results in close accord with observation.

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